Model Predictive Control (MPC)

Predictive, Optimization Based Planning Method

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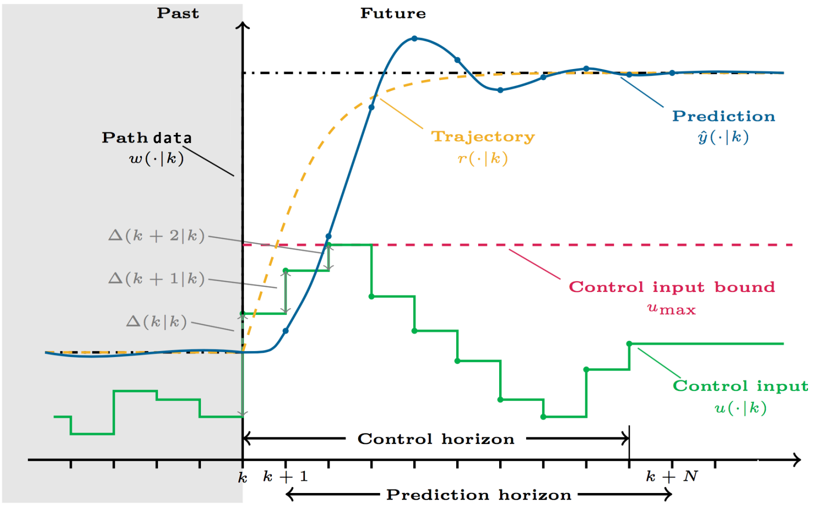
USAGE OF THE METHOD

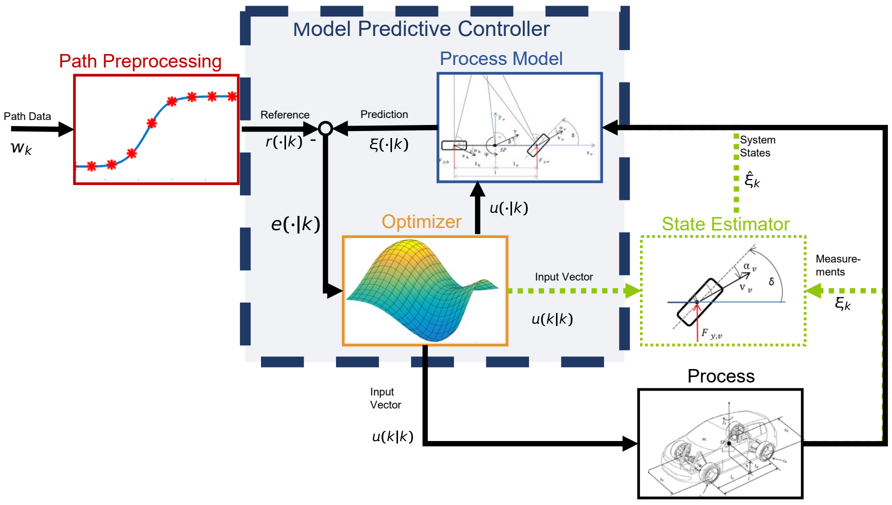
The approach determines a predictive and optimal controller output with explicit consideration of constrains based on a vehicle model (kinematic and dynamic). This is realized by utilizing iterative online optimization algorithms. The MPC approach is well established in industry, originally developed for solving control problems in the domain of process engineering [1].

PROBLEM STATEMENT

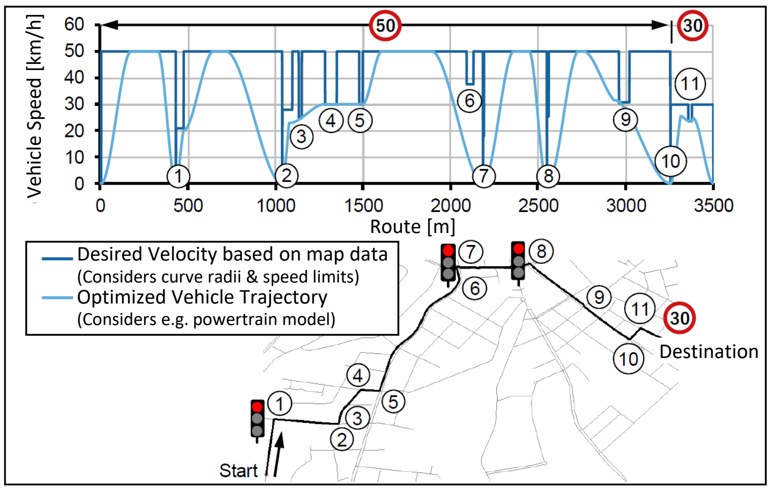
The **optimization scheme**, which is solved in each iteration is visualized in the following graphic.



The **Implementation** follows roughly the following concept.



The MPC approach is not only limited for classical path following tasks. More sophisticated tasks like Green light optimal speed advisory [2,3], Collison avoidance [4], etc. can be easily realized.

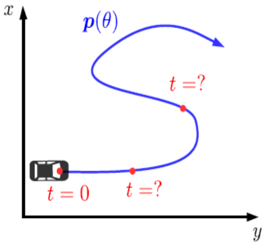


PATH-FOLLOWING CONTROL FOR AUTOMATED DRIVING [5,6]

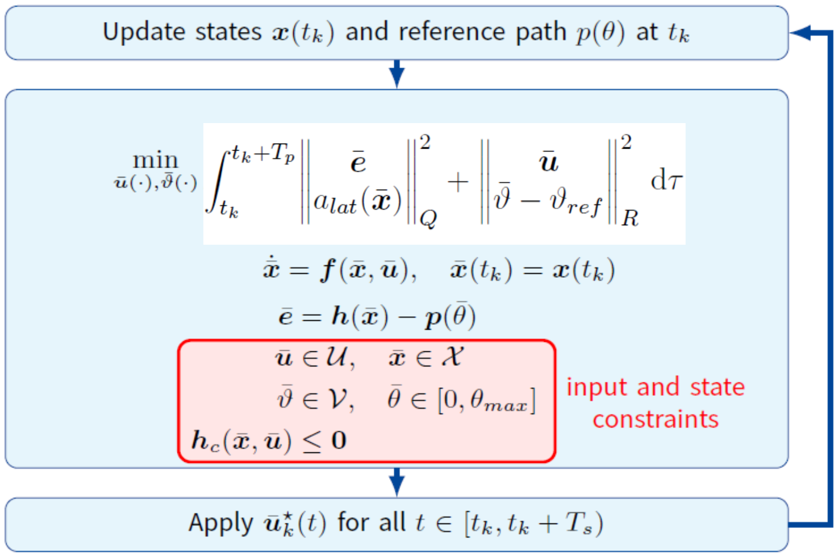
General Task

- Steering a vehicle autonomously along a given reference path
- Velocity along the reference path is not fixed a priori. When to be where is not predefined. This is typical for a path-following problem.

1. Path convergence: The system output $\boldsymbol{y}$ converges to the set $\mathcal{P}$ such that $\lim_{t\to\infty} \|\boldsymbol{e}(t)\| = 0$.
2. Velocity convergence: The path velocity $\dot{s}(t)$ converges to a predefined evolution $\dot{s}_{ref}(t) \geq 0$ such that $\lim_{t\to\infty} \|\dot{s}(t) - \dot{s}_{ref}(t)\| = 0$.
3. Constraint satisfaction: The state and input constraints $\mathcal{X}$ and $\mathcal{U}$ are satisfied $\forall t \in [t_0, \infty)$.



Solution Strategy



Update states $x(t_k)$ and reference path $p(\theta)$ at t_k

$$\min_{\bar{u}(\cdot),\bar{\vartheta}(\cdot)} \int_{t_k}^{t_k+T_p} \left\| \begin{matrix} \bar{e} \\ a_{lat}(\bar{\boldsymbol{x}}) \end{matrix} \right\|_Q^2 + \left\| \begin{matrix} \bar{\boldsymbol{u}} \\ \bar{\vartheta} - \vartheta_{ref} \end{matrix} \right\|_R^2 \mathrm{d}\tau$$

$$\dot{\bar{\boldsymbol{x}}} = \boldsymbol{f}(\bar{\boldsymbol{x}}, \bar{\boldsymbol{u}}), \quad \bar{\boldsymbol{x}}(t_k) = \boldsymbol{x}(t_k)$$

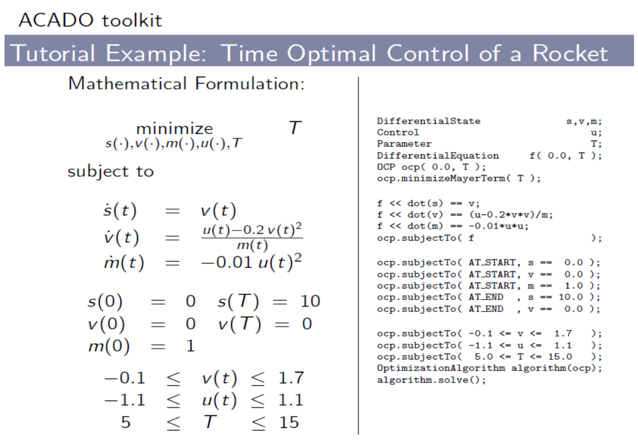
$$\bar{e} = \boldsymbol{h}(\bar{\boldsymbol{x}}) - \boldsymbol{p}(\bar{\theta})$$

$$\bar{\boldsymbol{u}} \in \mathcal{U}, \quad \bar{\boldsymbol{x}} \in \mathcal{X}$$
$$\bar{\vartheta} \in \mathcal{V}, \quad \bar{\theta} \in [0, \theta_{max}]$$
$$\boldsymbol{h}_c(\bar{\boldsymbol{x}}, \bar{\boldsymbol{u}}) \leq \boldsymbol{0}$$

input and state constraints

Apply $\bar{u}_k^\star(t)$ for all $t \in [t_k, t_k + T_s)$

Implementation Approach [acado.sourceforge.net]



ACADO toolkit

Tutorial Example: Time Optimal Control of a Rocket

Mathematical Formulation:

$$\underset{s(\cdot),v(\cdot),m(\cdot),u(\cdot),T}{\text{minimize}} \quad T$$

subject to

$$\begin{aligned} \dot{s}(t) &= v(t) \\ \dot{v}(t) &= \frac{u(t)-0.2\,v(t)^2}{m(t)} \\ \dot{m}(t) &= -0.01\,u(t)^2 \end{aligned}$$

$$\begin{aligned} s(0) &= 0 \quad & s(T) &= 10 \\ v(0) &= 0 \quad & v(T) &= 0 \\ m(0) &= 1 \end{aligned}$$

$$\begin{aligned} -0.1 &\leq v(t) \leq 1.7 \\ -1.1 &\leq u(t) \leq 1.1 \\ 5 &\leq T \leq 15 \end{aligned}$$

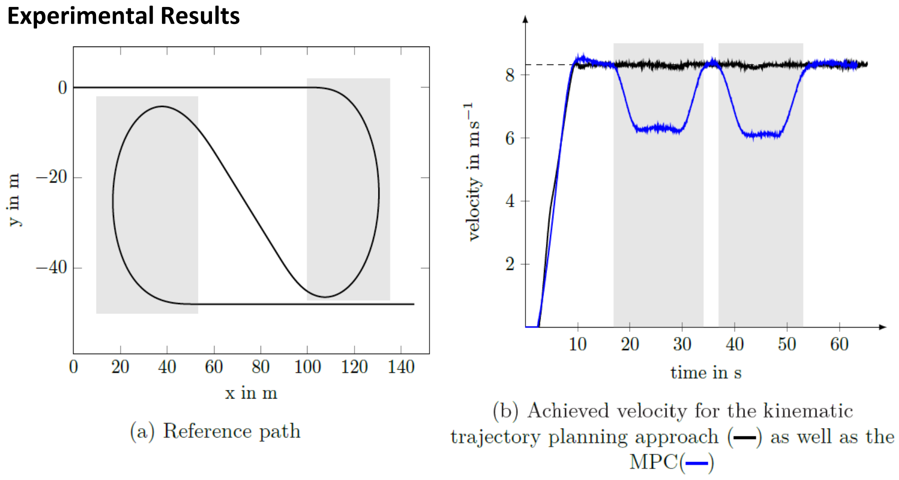
```
DifferentialState                s,v,m;
Control                              u;
Parameter                            T;
DifferentialEquation     f( 0.0, T );
OCP ocp( 0.0, T );
ocp.minimizeMayerTerm( T );

f << dot(s) == v;
f << dot(v) == (u-0.2*v*v)/m;
f << dot(m) == -0.01*u*u;
ocp.subjectTo( f                   );

ocp.subjectTo( AT_START, s ==  0.0 );
ocp.subjectTo( AT_START, v ==  0.0 );
ocp.subjectTo( AT_START, m ==  1.0 );
ocp.subjectTo( AT_END  , s == 10.0 );
ocp.subjectTo( AT_END  , v ==  0.0 );

ocp.subjectTo( -0.1 <= v <=  1.7   );
ocp.subjectTo( -1.1 <= u <=  1.1   );
ocp.subjectTo(  5.0 <= T <= 15.0   );
OptimizationAlgorithm algorithm(ocp);
algorithm.solve();
```

Experimental Results



(a) Reference path

(b) Achieved velocity for the kinematic trajectory planning approach (—) as well as the MPC(—)

REFERENCES
[1] J. Maciejowski. Predictive Control with Constraints. Prentice Hall. Harlow. 2002
[2] S. Übel, S. Kutter, K. Hipp, F. Schrödel. A Computationally Efficient MPC for Green Light Optimal Speed Advisory. International Conference on Vehicle Technology and Intelligent Transport Systems. 2019
[3] F. Schrödel, J. Gutsche, T. Alscher, R. Baumgart. Steigerung der Effizienz in hochautomatisierten Fahrzeugen durch optimierte Fahrstrategien. ATZ. 2019
[4] A. Katriniok: Optimal vehicle dynamics control and state estimation for a low cost GNSS-based collision avoidance system. Free online access. Dissertation. RWTH. 2014
[5] R. Ritschel, F. Schrödel, J. Hädrich, J. Jäckel. Nonlinear Model Predictive Path-Following Control in the Context of Highly Automated Driving. IFAC Symposium on Intelligent Autonomous Vehicles. 2019
[6] R. Voßwinkel, M. Gerwien, A. Jungmann, F. Schrödel. Intelligent decision making and motion planning for automated vehicles in urban environments. In: Autonomous driving and driver assistance systems. CRC Press (under review)